## GCE Examinations

## Advanced Subsidiary

## Core Mathematics C2

## Paper E

Time: 1 hour 30 minutes

## Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.
Mathematical formulae and statistical tables are available.
This paper has nine questions.

## Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

1. Evaluate $\quad \int_{2}^{4}\left(2-\frac{1}{x^{2}}\right) \mathrm{d} x$.
2. 

$$
\begin{equation*}
\mathrm{f}(x)=x^{3}+4 x^{2}-3 x+7 \tag{5}
\end{equation*}
$$

Find the set of values of $x$ for which $\mathrm{f}(x)$ is increasing.
3. Given that $p=\log _{2} 3$ and $q=\log _{2} 5$, find expressions in terms of $p$ and $q$ for

> (a) $\quad \log _{2} 45$,
> (b) $\quad \log _{2} 0.3$
4. The coefficient of $x^{2}$ in the binomial expansion of $(1+k x)^{7}$, where $k$ is a positive constant, is 525 .
(a) Find the value of $k$.

Using this value of $k$,
(b) show that the coefficient of $x^{3}$ in the expansion is 4375,
(c) find the first three terms in the expansion in ascending powers of $x$ of

$$
\begin{equation*}
(2-x)(1+k x)^{7} . \tag{3}
\end{equation*}
$$

5. (a) Write down the exact value of $\cos \frac{\pi}{6}$.

The finite region $R$ is bounded by the curve $y=\cos ^{2} x$, where $x$ is measured in radians, the positive coordinate axes and the line $x=\frac{\pi}{3}$.
(b) Use the trapezium rule with three equally-spaced ordinates to estimate the area of $R$, giving your answer to 3 significant figures.

The finite region $S$ is bounded by the curve $y=\sin ^{2} x$, where $x$ is measured in radians, the positive coordinate axes and the line $x=\frac{\pi}{3}$.
(c) Using your answer to part (b), find an estimate for the area of $S$.
6.


Figure 1
Figure 1 shows triangle $A B C$ in which $A C=8 \mathrm{~cm}$ and $\angle B A C=\angle B C A=30^{\circ}$.
(a) Find the area of triangle $A B C$ in the form $k \sqrt{3}$.

The point $M$ is the mid-point of $A C$ and the points $N$ and $O$ lie on $A B$ and $B C$ such that $M N$ and $M O$ are arcs of circles with centres $A$ and $C$ respectively.
(b) Show that the area of the shaded region $B N M O$ is $\frac{8}{3}(2 \sqrt{3}-\pi) \mathrm{cm}^{2}$.
7. The circle $C$ has the equation

$$
x^{2}+y^{2}+10 x-8 y+k=0
$$

where $k$ is a constant.
Given that the point with coordinates $(-6,5)$ lies on $C$,
(a) find the value of $k$,
(b) find the coordinates of the centre and the radius of $C$.

A straight line which passes through the point $A(2,3)$ is a tangent to $C$ at the point $B$.
(c) Find the length $A B$ in the form $k \sqrt{3}$.
8. Amy plans to join a savings scheme in which she will pay in $£ 500$ at the start of each year.

One scheme that she is considering pays $6 \%$ interest on the amount in the account at the end of each year.

For this scheme,
(a) find the amount of interest paid into the account at the end of the second year,
(b) show that after interest is paid at the end of the eighth year, the amount in the account will be $£ 5246$ to the nearest pound.

Another scheme that she is considering pays $0.5 \%$ interest on the amount in the account at the end of each month.
(c) Find, to the nearest pound, how much more or less will be in the account at the end of the eighth year under this scheme.
9. The polynomial $\mathrm{f}(x)$ is given by

$$
\mathrm{f}(x)=x^{3}+k x^{2}-7 x-15,
$$

where $k$ is a constant.
When $\mathrm{f}(x)$ is divided by $(x+1)$ the remainder is $r$.
When $\mathrm{f}(x)$ is divided by $(x-3)$ the remainder is $3 r$.
(a) Find the value of $k$.
(b) Find the value of $r$.
(c) Show that $(x-5)$ is a factor of $\mathrm{f}(x)$.
(d) Show that there is only one real solution to the equation $\mathrm{f}(x)=0$.

## END

